

Concrete: Computation and Optimization

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Abstract

New materials require new design and construction methods. Even old materials are being continually developed with new properties that challenge the way we use them. A recent cycle of innovations has led to concretes with considerable and effective elastic limit in tension and flexural strength. The possibility to design in concrete as a single orthotropic material with both tensile and compressive properties create an opportunity for new products but also require new design approaches.

Topology optimization as an architectural design tool is largely unexplored, in contrast to its wide use in the field of mechanical engineering. Topologically optimized shapes are fundamentally different from standard structural shapes and require highly customized means of fabrication. The resulting members can be lighter, use less material, yet still be as strong. Perhaps of greatest importance is the observation that the topologically optimized shape simultaneously manifests a structural optimum and an emergent aesthetic.

This presentation will introduce the basics of structural topology optimization, existing software, and show how it was used in architectural technology coursework. The assignment in view, given to intermediate architectural students, is to design and optimize a structural beam and to subsequently fabricate it in ultra-high-performance concrete using consumer level CNC-milling of polystyrene casting formwork. Computer stress simulations were compared to physical crush tests.

An increasing number of architects and engineers are well-versed in emerging digital fabrication and computation technologies. The presentation will posit that

the materials with emerging properties and accessible computation tools provide a platform for both architects and engineers to engage in the problem of combining structural efficiency and aesthetic.

Keywords: Computational analysis, optimization, Pedagogy, Concrete, Fabrication

Integration of Aesthetics and Structural Engineering

Architecture and structural engineering are professions that have a historically close relationship. Today, however, a common sentiment is that architects contribute attractive yet costly solutions and engineers are considered of a dull and practical mindset. A main point of distinction between the two disciplines is the issue of cost¹. While often a secondary consideration for architects, economy is one of the central goals of structural design. Great works of structural engineering integrate economy, efficiency, and elegance². Designers who successfully integrated aesthetics and structure, like Robert Maillart, Pier Luigi Nervi, Gustave Eiffel, John Roebling, and Felix Candela, demonstrated a focus on low cost by also integrating a thoughtful or innovative approach to construction in their works³. Now professionals on either side of the architecture/engineering divide see the other as superfluous to their design process. This diminished respect for each other may in part be due to the decreasing emphasis on structures in architectural education. For instance, in 1965 architecture master's students at Yale were required to take six semesters of structures courses. Those were reduced to three in 1975, and two in 1999⁴. Similarly, there is a lack of instruction on aesthetics and design history in modern engineering

curricula, whose accreditation criteria do not include any mention of ethics or aesthetics⁵. The wide adoption of digital technologies in the AEC professions gives rise to the opportunity for both architects and engineers to be effectively equipped to share the building design realm in both structural and aesthetic terms. An increasing number of architects and engineers are well-versed in emerging digital fabrication and computation technologies. The ease of use and accessibility of topology optimization tools provide a platform for both architects and engineers to engage in the problem of combining structural efficiency and aesthetics. It is now possible on a given project with typical time constraints to evaluate many more design proposals and gain much deeper insights into theoretical concepts than ever before.

Introduction to topology optimization

Topology optimization is a computational process by which a surface or a volume of a member under load is subdivided in a number of finite small areas or volumes, called *finite elements*. Each finite element is assigned a density that corresponds to the density of a structural material, such as concrete or steel. A density of zero would signify a void. In the beginning of the optimization process all finite elements in the body are given the same starting density, but during the optimization sequence densities are distributed according to the optimization objectives – in structures, that objective could be to maximize the stiffness of the member under load while taking into account the mechanical properties of the material. At a chosen end of the optimization process structural material is redistributed and a new *optimized* topology is generated.

The ultimate goal of topology optimization is to find the best structural layout, or material distribution of a structure, to fulfill its function in an optimal manner while fulfilling a set of behavior constraints early in the design stage⁶. The conventional approach to incorporating

structural considerations in pre-design such as desired shape, size, and strength is to parameterize an existing design and find its best fit. Usually this process limits the design outcomes to the choice of precedents and the creativity of the designer. Topology optimization as an early design tool dramatically expands the design possibilities. The optimization algorithm presents to the designer's evaluation a wide array of design features, such as overall shape of the structure, the location, shape and size of holes, supports, etc.

Significance of topology optimization

Topologically optimized shapes are fundamentally different from standard structural shapes, which are derived from casting or extrusion methods of fabrication and assume a degree of structural redundancy. In comparison, topologically optimized shapes require highly customized means of fabrication and the resulting members can be lighter, use less material, yet still be as strong. Perhaps of greatest importance is the observation that the topologically optimized shape simultaneously manifests a structural optimum and an emergent aesthetic⁷. Topology optimization as an architectural design tool is largely unexplored, in contrast to its wide use in the field of mechanical engineering. Present mass-customizable fabrication technologies, such as CNC-milling, vacuum forming, and 3d-printing, make the wider deployment of topologically optimized architectural and structural members economically viable. As a design approach, topology optimization holds a significant potential for design innovation and can lead to novel structural morphologies that transcend classical typological classification.

The offices of Skidmore, Owings & Merrill (SOM) are leaders in reinforcing the trans-disciplinary collaborations between architects and engineers. Their increased use of optimization algorithms and visualization of the flows of forces give architects a powerful intuitive understanding of the distribution of stresses and magnitudes of

displacements, which in turn informs decisions about how the overall shape of the buildings affects its structural frame.

*SOM designers and engineers have found that, like the graphic statics analytical methods conceived decades earlier, the visualization of the structural forces ... can often lead designers to possible design solutions which can be directly inferred from the visualizations*⁸.

Examples of large-scale implementation of structural topology optimization are the tower projects in the TransBay Transit Center in San Francisco and Shanghai Center in Shanghai (both 2010) where the optimization process iteratively redistributed a fixed amount of structural material in order to realize the most efficient use of that material. More notably, for the Commercial development project, Shanghai, China (2011) topology optimization revealed a novel way in which the multi-span bridge element connects three towers – the irregular pattern for an optimal structural system for the bridge component of this project was incorporated as part of the architectural tectonics.

Recent analysis of Catalan and Guastavino domes, carried out by John Ochsendorf at MIT, utilizes a combination of graphic and finite element optimization models, while the continued construction and reconstruction of Gaudi's Sagrada Familia is another great example of advanced application of structural topology optimization tools.

Method

There are multitudes of approaches to computing topology optimization, more popular among which are *homogenization-based*, *power-law*, and *evolution-based*⁹. While most approaches have found useful and established application in mechanical engineering, few have found consumer-level applications. The TopOpt plugin for the NURBS modeling program *Rhinoceros*®¹⁰ and its compendium parametric design module

Grasshopper™¹¹ utilizes an optimization procedure based on the paper “A 99-line topology optimization code written in MATLAB” by Ole Sigmund¹². TopOpt is written by the TopOpt research group at the Technical University of Denmark (DTU) and is one of few tools that are specifically geared towards designers, engineers and architects who experiment with design-related methodology and research¹³. One feature of the TopOpt procedure is the ability to interactively configure the optimization setup, such as supports, loads, solids and voids, while the optimization is in progress. Other interesting features is the inclusion of specific procedures that allow for tension and compression prioritization of a single material. These features make the software extremely versatile for analytical experimentation with single linear-elastic orthotropic materials. An obvious material application for this feature is concrete, for which the optimization routine should prioritize load-carrying capability in compression.

There are a number of software packages available on the market that compare to TopOpt. *SolidThinking Inspire*, *Abaqus Topology Optimization Module (ATOM)*, *Tosca Structure*, and *Nastran* are among the more popular. What sets TopOpt apart are two important characteristics: for simpler shapes and loading conditions TopOpt requires minimal set-up and the optimization routine is carried out relatively fast. A limitation to its wider applicability is that it is not well suited for working with more complex and irregular shapes under varied loads. This was deemed of no consequence for the goals of this study. What distinguished TopOpt in our view was that the interface allowed for interactive changes of the design parameters while the optimization was still in process – the designer does not need to wait until the optimization routine is complete before decisions on new optimization parameters can be made. Its speed and interactivity allowed us to almost instantaneously get feedback on design decisions and change the direction of the optimization in nearly real-time.

Concrete and Topology Optimization

The predominant model of analysis of concrete shapes is the so-called *strut-and-tie* model and was initially developed in the late 1800s by Wilhem Ritter and Emil Moersch. The strut-and-tie model of analysis assumes reinforced concrete (RC) beams, for instance, to exhibit truss-like behavior. This truss analogy provides a convenient visualization of the flow of forces and identified steel locations. Extensive research in support of the RC truss model has led to its prevalent method of structural analysis and its inclusion in the *Canadian Concrete Design Code* (1984), the *AASHTO* bridge code (1994), and the *American Concrete Institute* (2002) building code. The free form nature of topology optimization, however, enables the discovery of solutions with higher efficiency that are not straight and appear organic. These solutions tend to be complex, requiring curved rebar or rebar with varying thickness¹⁴. Due to the highly diverse optimization patterns developed for the compressive material (concrete) and tensile material (steel) and their complex geometric relationship, many topologies are simply impractical to fabricate on a mass scale. Reinforced concrete is a complex composite material and no current topology optimization methods are capable of accounting for transverse tensile stresses that may develop in compression members caused by force-spreading. Current work on steel-reinforced concrete optimization focuses on the application of parallel models of analysis – an orthotropic material is assumed for concrete and the tensile stresses are assumed to be carried out by steel in a truss-like fashion. Rebar is therefore placed in linear segments, while the compressive loads within the concrete part are allowed to take any shape¹⁵.

In view of a growing body of research in allowing the selective application for compressive and tensile forces to separate structural materials that are in composite action with each other, our experiment does not aim to substitute standard methods of structural topology

optimization of reinforced concrete. Rather, we borrow optimization methods used in mechanical engineering with applications involving polymers and metal alloys and take advantage of emerging properties of concrete that allow us to treat it as an isotropic elastoplastic material with distinct compressive and tensile strengths.

Current developments in ultra-high-performance concrete have challenged the traditional assumptions associated with concrete. For instance, a common ultra-high performance concrete (UHPC) product currently on the market, has an elastic limit in tension of up to 10 MPa (1,450 psi) and flexural strength of up to 40 MPa (5,800 psi)¹⁶, while compressive strength can run up to 200 MPa (29,000 psi)¹⁷. As a comparison, normal strength Portland cement concrete, which is commonly used in residential structural construction, has an average tensile strength of 3.5 MPa (500 psi), an average flexural strength of 4 MPa (580 psi), and an average compressive strength of 30 MPa (4,300 psi)¹⁸. The possibility to apply both tensile and compressive properties to a single orthotropic material make UHPC particularly suited for TopOpt's TenCom.1Mat procedure.

Example

Topology optimization was carried out on a simply supported ultra-high-performance concrete beam with a uniformly distributed load, Fig. 1. This loading and support configuration can easily be analyzed and



Standard Beam (Case B)

Fig. 1 Standard beam, elevation (drawing not to scale)

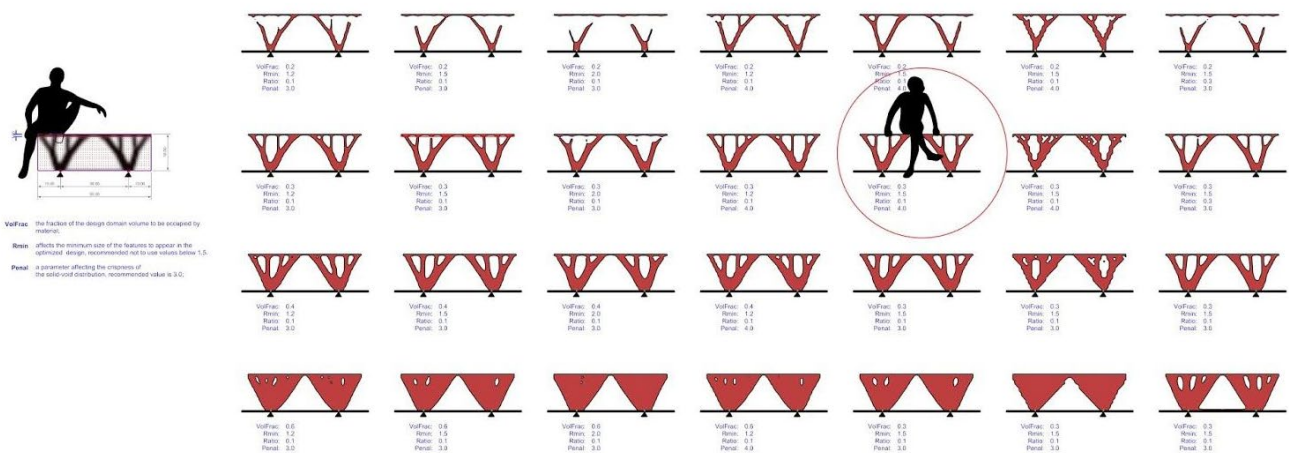


Fig. 2 A matrix of comparative studies (optimization results and values).

compared to existing shapes and common material properties. The beam was further modeled using two types of design spaces: the design space is the volume where the material density may be varied and through the process of optimization material may be removed; the non-design space is volume that is given constant density to be included in the calculations. However, it is excluded from optimization. In this way we have specified areas where material removal is undesirable, such as a deck, a seat, or a support ledge. A non-design 1" thick plate was assigned at the top.

In order to differentiate the performance of the optimized design a series of comparative studies were generated, Fig. 2. The following configuration inputs were variably adjusted: *VoIFrac* – the fraction of the design volume to

be occupied by material; *Rmin*- affects the minimum size of the features to appear in the optimized design; *Penal* – a parameter affecting the crispness of the solid-void distribution; *Ratio* – a parameter controlling the prioritization with respect to tension and compression.

The initial optimization objective was to minimize the deformation energy while achieving a 30% reduction of volume. In consecutive iterations, the varying constraints of input produce a matrix of topologies that contain both thick and thin parts, many of which would be difficult to fabricate. That difficulty can be alleviated by controlling the minimum size constraint, *Rmin*, and by varying the *VoIFrac* and *Penal* values. The final topology was chosen to reflect the flow of forces where, in the middle, a void is left by the formation of an arch, and increasing stress around the bases cause transverse webs to form, Fig. 3.

Numerical comparison

Two digital models were created using the finite element analysis software Abaqus CAE to compute the ultimate strength and quantify the efficiency of the optimized shape. “Case A” depicts the optimized shape created using TopOpt, and “Case B” depicts a standard rectangular beam shape with the same overall dimensions, support conditions, and material properties

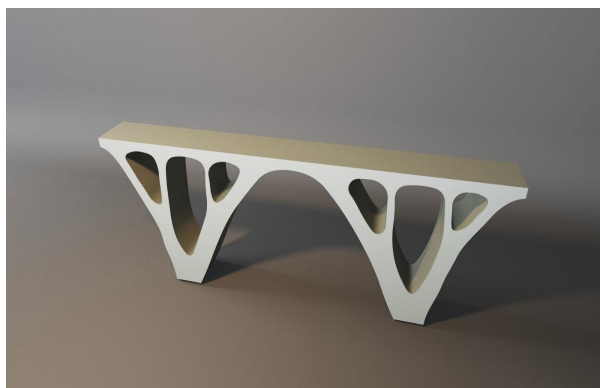


Fig. 3 Rendering of optimized topology chosen for fabrication

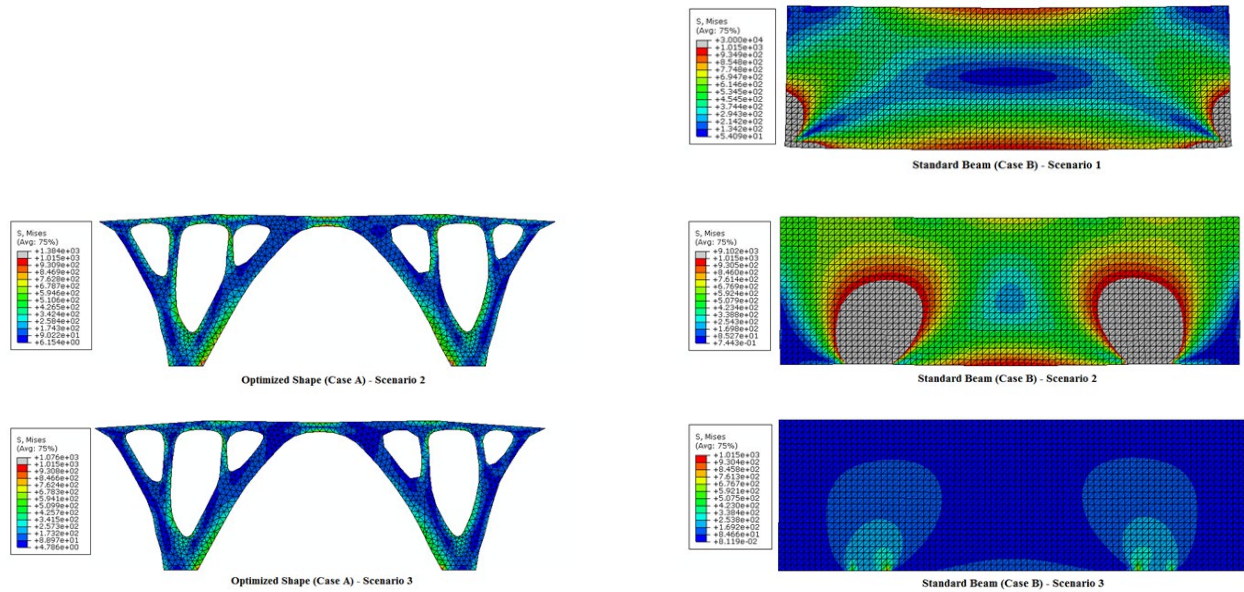


Fig. 4 Von Mises stress contours are shown in the figures for Case A and B for all 3 scenarios.

as the optimized shape. Volumes of Case A and Case B were automatically calculated with AutoCAD and found to be 0.037m^3 (2280 in³) and 0.131m^3 (8000 in³) respectively.

Both Case A and B had simply supported boundary conditions with a pin (horizontal and vertical translation fixed, allowing rotation) support at one end and a roller (vertical deflection fixed) support at the other end. Out-of-plane deformation was restricted in the 2D models created. A vertical uniform load was applied to the top face in each case and gradually increased until the maximum compressive or tensile stress was reached. Automated meshing with 3-noded linear plane stress triangles were used to create the mesh of both Case A and B. Element size was gradually decreased until approximately 16.5mm (0.65 in.) when results were no longer sensitive to the element size.

Each model was assigned material properties supplied by the UHPC manufacturer. As discussed above, the maximum compressive strength and maximum tensile strength for our particular material are approximately 130MPa (18,850psi) and 7MPa (1,015psi) respectively.

The unit weight is 24.5kN/m^3 (156pcf), which compares to normal strength, normal weight concrete at 23.6kN/m^3 (150pcf). To minimize computational effort, the material was assumed to have linear-elastic behavior up to failure in both compression and tension.

Three scenarios were considered for the construction of the numerical models. In scenario 1, the supports of Case B are placed at the end of the beam creating a span of 127cm (50in.). In scenario 2, the supports of Case B are identical to that of Case A. Both scenario 1 and 2 consider only a flexural failure occurring at the center of each model, where the bending moment will be highest, while scenario 3 considers the possibility of failure elsewhere in each model.

Von Mises stress contours are shown in the figures for Case A and B for all 3 scenarios, Fig. 4. Loaded stresses at each point in a body have a different value depending on location and direction. As a consequence, each finite point in a body has multiple stress components depending on the orientation of the point. Von Mises theory, also referred to as *the maximum distortion energy theory*¹⁹, is one of the most common methods to combine stress components to predict failure of a body. In

summary, the standard rectangular shape beam is stronger than the optimized shape; however, the standard beam is also significantly heavier with a weight of 3213N (722lb) compared to the optimized weight of 916N (206lb). The results for each scenario are provided in Table 1, where “Efficiency” is the ratio of the total applied load (the product of the uniform load and the beam length) and the weight of the shape. Due to the simplicity of the geometry and material properties, the Case B numerical result for scenario 1 was easily validated with an analytical calculation, which was found to be within 2%.

Table 1: Comparison of the strength and efficiency of the two cases and loading scenarios.

Scenario	Optimized (Case A) total weight 206 lbs		Standard (Case B) total weight 722 lbs	
	Max load kN/m(lbf/in)	Efficiency	Max load kN/m(lbf/in)	Efficiency
1	N/A	N/A	23.6 (135)	9.35
2	4.73 (27.0)	6.56	96.3 (550)	38.1
3	3.68 (21.0)	5.10	10.5 (60.0)	4.15

Cases 1 and 2 are assuming that flexural failure will occur at midspan of each section; however, due to the chosen length to depth ratio, flexural failure is unlikely. Therefore, case 3 is most reasonable to occur. The optimized shape is 22.9% more efficient than the standard shape.

Production of optimized forms and casting

The form for the chosen design is manufactured from polystyrene blocks which will be used as molds for casting concrete. The forms were cut on a CNC-router and assembled in a compressive frame. The form was sealed with primer and petroleum jelly, making it water- and air-tight. The concrete was mixed according to manufacturer’s ratios and mixing procedures. The concrete was cast and de-molded after 7 days and moisture-cured for 3 additional weeks, Fig. 5



Fig. 5 Image of CNC-milled polystyrene casting form

Conclusions

We have observed significant weight and strength difference between the standard and the optimized shapes. The optimized shape is 28.5% lighter and 35% weaker. However, the overall efficiency, as represented by a strength to weight ratio, is significantly in favor of the optimized shape. The optimized shape is 22.9% more efficient.

The following preliminary conclusions were made. Topology optimization:

- May lead to the development of new structural shapes for fiber-reinforced concrete
- May lead to significant reduction in material use.
- May achieve comparable to standard shapes strength, however there is a relation between the allowable strength to the increased ability to experiment with formal topology
- Allows for direct correlation between aesthetic characteristics and structural performance

Another observation was that the existing commercially available software can be used in optimizing structural members.

In addition to the application of the optimization routine on a simply supported beam, the team plans to test the approach on larger structural beams. We are preparing a case study that compares conventional precast AASHTO



Fig. 6 Photograph of cast beam. Overall dimensions: 50" (length) x 10" (width) x 16" (depth); weight: 206 lbs

beam to the potential gains in structural economy of an optimized beam. By illustrating the expressive potential of structurally optimized precast members we hope to be able to introduce a strictly architectural agenda in structural design. A current call for proposals to the

National Science Foundation specifically invites participation from architects in the area of topology optimization. This introductory work and its dissemination are an important step in securing funding and furthering the line of inquiry.

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¹⁰ Rhinoceros is a trademark of Robert McNeel and Associates

¹¹ Grasshopper is a visual programming language developed by David Rutten at Robert McNeel & Associates

¹² In addition to Rhinoceros, TopOpt is available for Linux, Android, iOS, and Java3D (<http://www.topopt.dtu.dk/?q=node/792>, accessed on March 17, 2014). A similar to Top Opt tool is BESO2D (<http://www.rmit.edu.au/browse!ID=vxbyafpheur>, accessed on March 17, 2014)

¹³ Other commercial TO software available on the market are Autodesk Solidworks, geared towards industrial and product designers, Simulia Abaqus, with robust aerospace and automotive applications, FE-Design Tosca, with uses in additive manufacturing, and Altair OptiStruct in product and automotive design.

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