Logical transductions are not sufficient for notational equivalence

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1 Introduction

In recent years, the application of finite model theory to the study of phonological patterns and representations has been extremely fruitful. Strother-Garcia (2019) provides both a thorough study of the computational nature of syllabic processes and a formal comparison of several theories of representing syllabic structure[1], showing how one structure can be translated into another "without loss of information" (Strother-Garcia, 2019:3). While Strother-Garcia (2019) does specify that this notational equivalence holds "in a strict mathematical sense" (Strother-Garcia, 2019:72), Oakden (2020) and Danis & Jardine (2019); Jardine et al. (2021) extend this argumentation to having direct consequences for phonological theories.

Logical transductions are an important and enlightening tool in the investigation of phonological processes and representations, and the aim here is to define a program of formalizing and marrying the traditional expectations and intentions of phonological structure (including what linguistically relevant information is meant to be conveyed) within the strictly formal logical comparisons of theories. Despite the negative nature of the title of this work, it should be interpreted as "necessary but not sufficient". By expanding upon what is strictly mathematically true by virtue of the existence of a given transduction, into what precisely is relevant for linguistic theory, and how such representations are intended to work when implemented in a given theoretical framework or analysis, the hope is that these and any future conclusions are relevant for both a wide range of phonological applications, mathematical or otherwise.

Specifically, it is shown here that two models of segmental representation can be quantifier-free bi-interpretable (sharing the benchmark used in Strother-Garcia (2019) and Oakden (2020)), *while still predicting different sets of natural classes*. Since it is difficult to find a proposed phonological framework that does *not* operate over natural classes, natural class behavior serves as a clear case of information that is potentially *not* shared between two models that are otherwise shown to be logically equivalent, while still considering the representations in a theory-general way that is not dependent on any specific system of computation.

Additionally, because the definition of *strong generative capacity* in Miller (2001) is also housed within formal model theory, this framework provides a directly applicable way to define such a capacity for phonological theory, a notion that has been somewhat elusive otherwise (Dolatian et al., 2021). Specifically, natural class behavior is just one of many *interpretation domains* through with two representations can be compared, in addition to other phonologically relevant aspects (such as contrast preservation as defined in Oakden (2020)) beyond simply the their abstract formal structures.

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1 Strother-Garcia (2019) also serves as an excellent 'how-to' guide for applying model theory in phonology, and is highly recommended for any curious reader.

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2 Case study: place of articulation features

This section defines two models of segmental representation, differing in how they represent the major place of articulations of consonants and vowels. The models here are called **unified** and **v-features**.

The **unified** model is based on the fundamental idea in Sagey (1986), Clements (1991), Hume (1994), Clements & Hume (1995) and similar proposals, where (at least some) place feature labels for consonants and vowels overlap, and it is their specific geometry that determines their phonetic realization as either consonantal or vocalic stricture. By design, these proposals are indeed meant to group consonants and vowels together in specific ways: front vowels and coronal consonants are both [coronal], rounded vowels and consonants with closure at the lips are both [labial], and so on. The phonetic implementation of these place features depends on their position in the larger geometry: here, whether they are dominated by a C-place or V-place node.

The **v-features** model is based on those where the set of place features for vowels, such as [front], [back], and [round] features, are largely disjoint from the place features for consonants. Versions of this idea are common, and some examples are Chomsky & Halle (1968), Ní Chiosáin & Padgett (1993), Halle et al. (2000), among many others. Certain proposals, such as Odden (1991), take the additional step of reserving the [dorsal] feature for vowels only, and introduces the place feature [velar] for consonants that would otherwise be dorsal. Again, by design, the proposals serving as the inspiration for the v-features model made explicit decisions on what segments could be a natural class with others.

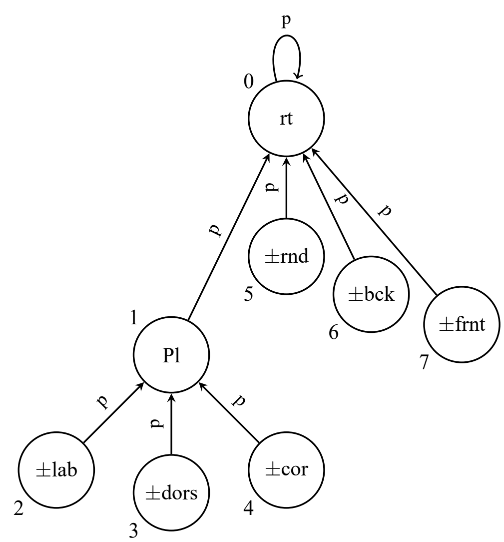
The models here are simplified versions of these theories meant to highlight this one crucial difference. These are not meant to be fully realized models of representation, but are meant to explicate these differences in a way that is still linguistically robust. Due to this, other areas of contrast, like features for vowel height, manner, laryngeal specification, and so on, are abstracted away from and not considered here. Their omission does not change the central point.

The two representations here are each treated as finite models. The signature of a finite model is a tuple of a domain of elements $\mathcal{D}$, a set of relations $\mathcal{R}$, and a set of functions $\mathcal{F}$. The domain is simply a set of unlabeled, unordered positions that are (potentially) given labels and positions through the definitions of any relations or functions. Additionally, a set of logical axioms (sentences) are defined for a model, such that every licit structure in that specific model should satisfy all axioms. In linguistic terms, since both representation models are fully specified geometries, the general axiom assumed for both is every segment in the inventory must contain the same relations in the fully specified example, and differ only on the $+$ or $-$ version of the feature labels (formal unary relations modeling linguistic binary features). See Strother-Garcia (2019) for a fuller and in depth definition of model theory as it is implemented here.

The models are also defined using the total, unary function $parent(x)$, where the output is the position y that is defined as the *parent* of x. Parenthood is domination within the tree structure and here is essentially the autosegmental association relation. This is implemented as a unary function rather than a binary relation in order to define the transduction using quantifier-free logic, which is an existing benchmark for similar studies.

2.1 *The v-features model* The v-features model represents a simplified version of those representational theories where the place features for vowels are largely separate from the place features for consonants. In the v-features model specifically, vowel place is controlled by the features [round], [back], and [front], while consonant place is trifurcated over [labial], [dorsal], and [coronal]. Central vowels are assumed to be [$-$front] and [$-$back], and a segment with [$+$back] and [$+$front] values is ruled out axiomatically. The three consonantal place features are dominated by an intermediate place node, though this node is not crucial for the current argument. Each feature is (linguistically) binary and all segments are fully specified for either $+$ or $-$ for each feature. The full structure is shown in Figure 1.

Figure 1: Fully-specified v-features model

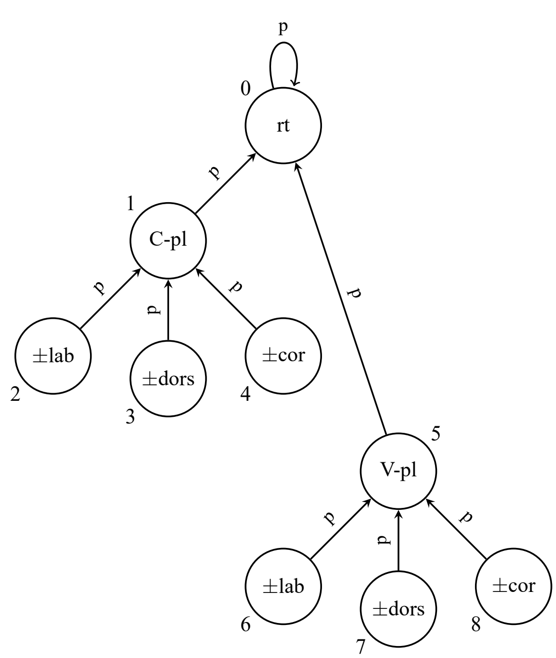


2.2 *The unified model* The unified model utilizes a place node for both consonant and vowel features, and is based on the general design principles of Clements & Hume (1995), stated below:

> As far as vocoids are concerned, rounded vocoids are [labial] by these definitions, front vocoids are [coronal], and back vocoids are [dorsal]. (Clements & Hume, 1995:227)

By design, this model creates natural classes between these groups of segments. The implementation into a formal model only allows us to precisely compare and enumerate the natural class predictions between models. The fully specified structure is given in Figure 2.

Figure 2: Fully-specified unified model



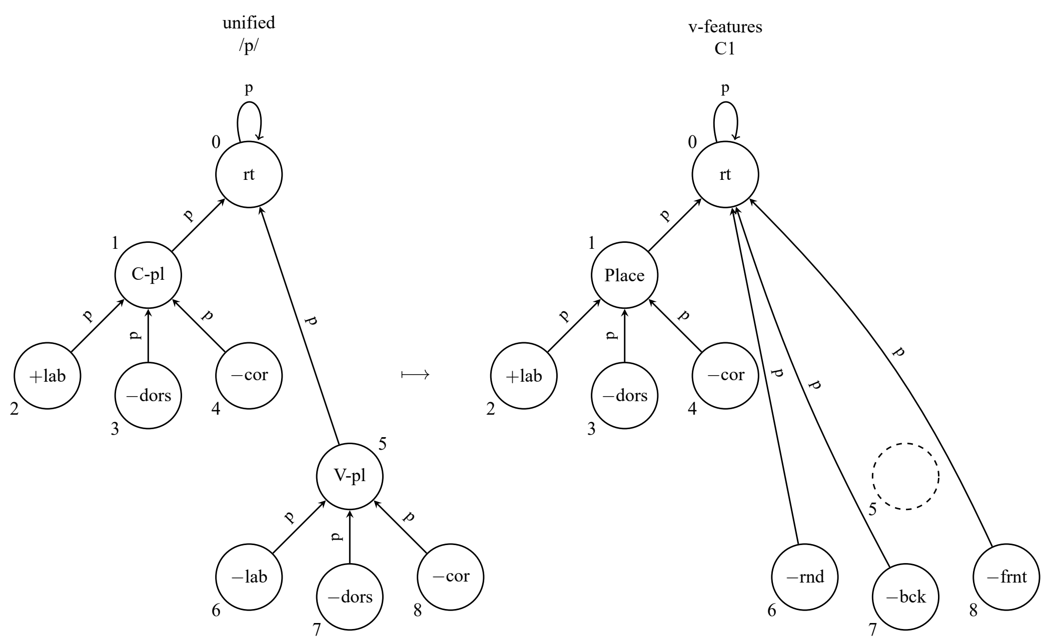
Both the v-features model and the unified model account for the same set of contrasts, or segmental distinctions. In terms of Oakden (2020), these models are then *contrast preserving*.

2.3 *Quantifier-free bi-interpretability* The two models are bi-interpretable under a quantifier-free transduction, as the following section shows. The transduction is quantifier-free because every structure in one can be translated into a structure in the other using a set of instructions (or rules) no more powerful than quantifier-free first order logic. The transduction is bi-interpretable because rules are defined in both orders, from unified to v-features and vice versa.

Quantifier-free bi-interpretability is noted as a nontrivial benchmark for linguistic equivalence in Strother-Garcia (2019) and Oakden (2020), and so that is the logical limit chosen for the transduction here.

2.3.1 *unified → v-features* The first direction of the transduction builds a structure in **v-features** starting with a structure in the **unified** model. The graphs in Figure 3 show this transduction applying to the structure for the segment /p/ specifically, though the full rules of the transduction (Figure 4) are general, so they account for any and all possible structures.

Figure 3: Transduction of /p/ from unified to v-features



$$\texttt{rt}(x^1) := \texttt{rt}(x) \tag{1}$$
$$\texttt{Place}(x^1) := \texttt{C-place}(x) \tag{2}$$
$$+\texttt{lab}(x^1) := +\texttt{lab}(x) \wedge \texttt{C-place}(\textit{parent}(x)) \tag{3}$$
$$+\texttt{cor}(x^1) := +\texttt{cor}(x) \wedge \texttt{C-place}(\textit{parent}(x)) \tag{4}$$
$$+\texttt{dors}(x^1) := +\texttt{dors}(x) \wedge \texttt{C-place}(\textit{parent}(x)) \tag{5}$$
$$-\texttt{lab}(x^1) := -\texttt{lab}(x) \wedge \texttt{C-place}(\textit{parent}(x)) \tag{6}$$
$$-\texttt{cor}(x^1) := -\texttt{cor}(x) \wedge \texttt{C-place}(\textit{parent}(x)) \tag{7}$$
$$-\texttt{dors}(x^1) := -\texttt{dors}(x) \wedge \texttt{C-place}(\textit{parent}(x)) \tag{8}$$
$$+\texttt{round}(x^1) := +\texttt{lab}(x) \wedge \texttt{V-place}(\textit{parent}(x)) \tag{9}$$
$$+\texttt{front}(x^1) := +\texttt{cor}(x) \wedge \texttt{V-place}(\textit{parent}(x)) \tag{10}$$
$$+\texttt{back}(x^1) := +\texttt{dors}(x) \wedge \texttt{V-place}(\textit{parent}(x)) \tag{11}$$
$$-\texttt{round}(x^1) := -\texttt{lab}(x) \wedge \texttt{V-place}(\textit{parent}(x)) \tag{12}$$
$$-\texttt{front}(x^1) := -\texttt{cor}(x) \wedge \texttt{V-place}(\textit{parent}(x)) \tag{13}$$
$$-\texttt{back}(x^1) := -\texttt{dors}(x) \wedge \texttt{V-place}(\textit{parent}(x)) \tag{14}$$
$$\textit{parent}(x^1) := (\textit{parent}(x))^1 \Leftrightarrow \neg\texttt{V-place}(\textit{parent}(x)) \tag{15}$$
$$\textit{parent}(x^1) := (\textit{parent}(\textit{parent}(x)))^1 \Leftrightarrow \texttt{V-place}(\textit{parent}(x)) \tag{16}$$

Figure 4: Transduction from unified to v-place

Since there are fewer nodes in the resulting v-features structure than in unified (as there is nothing corresponding to the V-place node), only a single copy set is needed. Rule (1) copies the root node label exactly in the same position. Rule (2) renames the C-place node to simply Place; this is done to align with the conventions in the literature; leaving the node labeled as C-place would not affect any of the results here.

Rules (3–8) preserve the labels for labial, coronal, and dorsal features *if the parent of these nodes in unified has the label of C-place*. Thus, by making use of the parent function in the transduction rules, we can look up one level to determine if a given place feature is meant to be interpreted as consonant or vocalic place.

Rules (9–14) crucially change the labels of these nodes into the labels (features) unique to the v-features model. Again the parent node labels are accessed, and if the parent node label is V-place, the labels [labial], [coronal], and [dorsal] are changed to [round], [front], and [back], respectively and preserving the $+/-$ values as well. Though these labels can still preserve the same contrasts of unified, it is the changing of labels in these rules that ultimately leads to each model predicting different natural classes of segments.

Finally, the parent function is defined for v-features based on the definition in unified. Rule (15) essentially states that all nodes in v-features will have the same parent function definition as the nodes in unified as long as the node in unified does not have a parent with the label V-place. If a node in unified does have the label of V-place, its corresponding node in v-features will be the parent of the parent node, as indicated by the nesting function definition in rule (16). This will "skip" the undefine node in v-features that corresponds to the position of the V-place node in unified.

2.3.2 *v-features $\rightarrow$ unified* The transduction from the v-features model to unified works similarly. The nodes with the labels representing labial, coronal, and dorsal features are copied directly, and the Place node gets relabeled C-place. However, since the unified model has a V-place node which v-features lacks, the transduction must make this extra node position available. A second copy set is assumed, indicated with the superscript 2 in the transduction rules. The node in copy set 2 that has the same position as the root node in the input gets the V-place label, and the parent function is defined accordingly. This is shown visually in Figure 5, with the full rules given in Figure 6.

Figure 5: Transduction of /p/ from v-features to unified

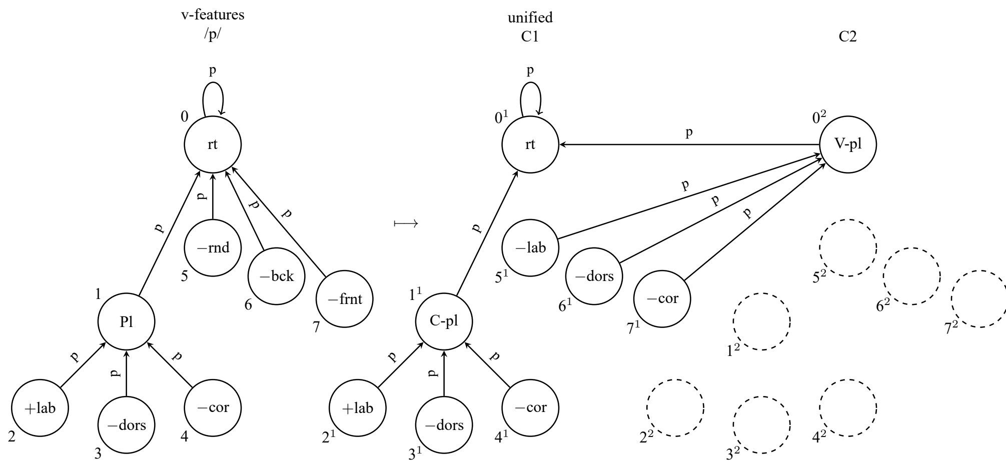


Figure 6: Transduction from v-features to unified

$$\texttt{rt}(x^1) := \texttt{rt}(x) \tag{17}$$
$$+\texttt{lab}(x^1) := +\texttt{round}(x) \vee +\texttt{lab}(x) \tag{18}$$
$$+\texttt{cor}(x^1) := +\texttt{front}(x) \vee +\texttt{cor}(x) \tag{19}$$
$$+\texttt{dors}(x^1) := +\texttt{back}(x) \vee +\texttt{dors}(x) \tag{20}$$
$$-\texttt{lab}(x^1) := -\texttt{round}(x) \vee -\texttt{lab}(x) \tag{21}$$
$$-\texttt{cor}(x^1) := -\texttt{front}(x) \vee -\texttt{cor}(x) \tag{22}$$
$$-\texttt{dors}(x^1) := -\texttt{back}(x) \vee -\texttt{dors}(x) \tag{23}$$
$$\texttt{C-place}(x^1) := \texttt{Place}(x) \tag{24}$$
$$\texttt{V-place}(x^2) := \texttt{rt}(x) \tag{25}$$
$$\textit{parent}(x^1) := (\textit{parent}(x))^1 \Leftrightarrow \neg\text{vowelFeature}(x) \tag{26}$$
$$\textit{parent}(x^1) := (\textit{parent}(x))^2 \Leftrightarrow \text{vowelFeature}(x) \tag{27}$$
$$\textit{parent}(x^2) := x^1 \Leftrightarrow \texttt{rt}(x) \tag{28}$$

Rule (18) copies the label of the root node into copy set 1. Notice that the next six rules, (18–23), all contain disjunctions. Each rules represent the “collapse” of two natural classes in v-features to a single one in unified: nodes labeled [+round] and nodes labeled [+labial] are *both* relabeled to just [+labial], and so on. The distinction between consonantal and vocalic place is not lost, however, as the combination of rule (25) along with the new definition of the parent function in (26–28) preserve and shift this distinction to the responsibility of the V-place node. As each representation is fully specified, the node in copy set 2 corresponding to the position of the root node will always get the label of V-place, but other nodes will only have this new V-place node as a parent iff they had any of the labels [round], [back], and [front] in the input structure (regardless of feature value). The function definition contains the user-defined predicate vowelFeature(x), which is defined below:

(29) $\text{vowelFeature}(x) = +\texttt{round}(x) \vee +\texttt{front}(x) \vee +\texttt{back}(x) \vee -\texttt{round}(x) \vee -\texttt{front}(x) \vee -\texttt{back}(x)$

Thus, even though rules of (18–23) collapse the distinction between vowel and consonant place, the eventual parenthood status of the V-place node preserves it based on which nodes had labels representing vowel place features in the input structure.

3 The models are not natural class preserving

In order to formally compare the predicted sets of natural classes between models, the following definitions hold. A **segment** is the fully specified structure from that model representing a single speech sound. An **inventory** is the set of all predicted structures based on the definitions and axioms of that model. A **natural class** for some substructure S is all structures in a model that have substructure S and no structures that to not contain substructure S. This implementation follows both the spirit and letter as most implementations in existing phonological theory. Note that this definition of natural class is not (yet) language specific—the classes in question are enumerated from the full set of contrasts of that model.

This also means that the definition of a natural class for one model is simply a set of structures defined by that model, and any strictly set-theoretic comparison between two models will be largely disjoint. To actually compare the results of two different models, a mapping of structures to atomic symbols (essentially, IPA characters) is assumed as part of the definitions for each model. For every possible structure in the v-features model, for instance, a mapping is defined that relates the structure containing a root note dominating [+lab],[−round], and so on, like that in Figure 5, to the atomic symbol [p], and so on, in a way that surprises no linguist.

This mapping enables the direct comparison of contrasts and natural classes between theories. The **natural class extensions** of a model are the natural classes where each element is mapped to its atomic, IPA equivalent. Two models are thus **natural class preserving** if their natural class extensions are set-theoretically equivalent. For any natural class that can be defined in one model, there is an equivalent natural class that can be defined in the other model where the set defined as the mapping of the structures in these two sets to one containing simply atomic characters is equivalent.

The method used here of determining the predicted natural classes is simply a search via enumeration. Given a set of structures (segments) that constitute one model, every possible substructure of every possible segment is determined. Then, for each of these substructures (or factors), every segment *containing* that substructure is collected. The result is then a set of sets, each set hen defined as all segmental structures sharing some specific piece of substructure. This is implemented in python, and the full code and results are available on GitHub (`https://github.com/nickdanis/autosegx`).

By design, these two models are meant to have differing natural class behavior. The differences in intended natural classes predicted indeed survive into their formal implementation as finite models, and yet this does not preclude the models from being logically equivalent. Specifically, there are natural classes in the unified model that have no corresponding class in the v-features model. There are no natural classes predicted by v-features that are not predicted by unified. Additionally, there are 238 natural classes that are shared by both models. In this sense, the natural class extensions of v-features are a properly included in those for unified.

The natural class extensions that are present only in **unified** are enumerated below.

(30) Natural classes specific to the **unfied** model

	defining factor	natural class extension
a.	`+cor`	[i kʲ kt pʲ t tʲ tp tʷ tˠ y]
b.	`+lab`	[kp kʷ p pʲ pʷ pˠ tp tʷ u y ʉ]
c.	`+dors`	[k kʲ kp kt kʷ kˠ pˠ tˠ u ɯ]
d.	`−cor`	[i k kʲ kp kt kʷ kˠ p pʲ pʷ pˠ t tp tʷ tˠ u y ɨ ɯ ʉ]
e.	`−lab`	[i k kʲ kp kt kʷ kˠ p pʲ pˠ t tʲ tp tʷ tˠ u y ɨ ɯ ʉ]
f.	`−dors`	[i k kʲ kp kt kʷ p pʲ pʷ pˠ t tʲ tp tʷ tˠ u y ɨ ɯ ʉ]

Each set happens to be defined by a single node, which are the positive and negative values of each of the place features. The segments in (a), for instance, all have a +cor node somewhere in their structure, either dominated by C-pl or V-pl. This was the intended effect in Clements & Hume (1995). The three classes defined by the negative values of the feature have a somewhat unintuitive interpretation: the segment [t] is in the natural class defined by −cor, but because there is a −cor specification under the V-pl node as there is no secondary articulation. The treatment of place features as binary valued, and not privative, is simply to aid in the logic of the transduction.

4 Phonology cares

In logical terms, the unary relations on each node in these structures are simply labels with no deeper meaning beyond tracking what is distinct from what. In linguistic terms, they represent features which not only define some articulatory or acoustic instruction but also serve as the basic structure over which rules and processes operate. The notion of linguistic equivalence, and specifically natural class preservation, is only then relevant once these representational models are implemented in some grammatical or computational framework. If the eventual grammatical framework operates over natural classes, then a transduction must be considered beyond its strict logical conclusion in how natural class information is preserved between the two models.

As it so happens, many phonological theories do in fact care about natural classes.

> In view of this, if a theory of language failed to provide a mechanism for making distinctions between more or less natural classes of segments, this failure would be sufficient reason for rejecting the theory as being incapable of attaining the level of explanatory adequacy. (Chomsky & Halle, 1968:335)

In linear, rule-based phonology, partially specified feature bundles select all segments with those specifications as the triggers or targets of phonological processes. This is true even for more recent implementations, such as that in the formal, substance-free phonology of Volenec & Reiss (2020); Reiss (2021):

> This combinability of features allows phonology to construct complex symbols from an inventory of simple parts, and provides an explanation for the so-called natural class behavior—different structures can behave alike because they contain identical substructures. (Volenec & Reiss, 2020:22)

> In Logical Phonology (see section 3), rules refer to natural classes by definition: a statement that cannot be formulated in terms of natural classes is not a rule. (Volenec & Reiss, 2020:28)

Additionally, in constraint-based theories such as Optimality Theory (Prince & Smolensky, 1993/2004), while there is some freedom in the range of how constraints are defined for a given analysis, the default starting point for markedness constraints is that they target some specific piece of structure, and assign violations based on candidates containing this structure. It could be a single feature or a bundle or collection of features, depending on the rest of the theory. What this means for the argument here is that if there is a hypothetical assimilation process that is to be modeled under common OT assumptions, there are clear and tangible differences in how the given representation affects the constraints necessary to capture a potential process. With differences in the constraint set, differences in the predicted factorial typology are sure to follow.

Assume that there is a hypothetical assimilation process where /ku/ maps to [pu]. The velar stop [k] assimilates its place to labial, caused by the vocalic rounding on [u]. This is one of the cross-categorical assimilation processes that is naturally captured due to the design philosophy of the representation in Clements & Hume (1995) and thus the unified model here. If we assume an Agree constraint Lombardi (1999); Bakovic (2000) for each individual feature in the segmental structure, a simplified version of an analysis would look as follows. Under the Unified model, since [u] and [p] both share a +lab node, this candidate satisfies a singular Agree[labial] constraint.

(31) unified

/uk/	Agr[lab]	F
uk	* W	L
up		*

In the v-features model, [p] and [u] do not form a natural class and the same assimilation processes cannot be analyzed in the same way. Keeping constant the assumption that there is one Agree constraint for every individual feature in the segmental structure, the intended winner is harmonically bounded[2] as there is no singular constraint that does the work of Agree[labial] with the unified model.

[2] Of course this is not a fully defined analysis and therefore this is not a complete argument for harmonic bounding, but the general point stands that capturing this same mapping under these assumptions requires a different method than with the unified model.

(32) v-features

/uk/	Agr[lab]	Agr[rnd]	F
uk	L	* e	L
up	*	*	*

There is likely a definition of CON that does allow this mapping to be captured under the v-features model, but now both GEN and CON are differ between the two analyses—GEN simply due to different representations being generated and CON because of the different resulting Agree constraints plus whatever else is needed. As the factorial typology of an optimality theoretic analysis falls out directly from the definitions of GEN and CON, any analyst should fully expect different predictions to be made. These different predictions ultimately stem from the choice of one model over another, despite both being logically equivalent.

A parallel argument can be made for the treatment of similar assimilation processes in an autosegmental framework, where the assimilation process itself is spreading. If the trigger and target of an assimilation process do not share the relevant pieces of substructure, then assimilation as a process cannot be due solely to spreading. Again, if the choice of representational models to capture the same linguistic mapping results in the need for different mechanisms in the grammatical framework used, then the fact that these models are equivalent in some logical sense must be considered in tandem with their measurable linguistic differences as well.

5 Towards a strong generative capacity for phonology

Oakden (2020) framed much of the discussion between the feature geometries investigated there in terms of *contrast preservation*: two theories are contrast preserving iff they can model the same set of phonological contrasts. The argument here builds on this with *natural class preservation*: two models are natural class preserving iff they predict the same sets of natural classes (as defined here). Both of these properties can be defined independent of a given transduction and serve as possible *interpretation domains* in terms of Miller (2001) for the definition of strong generative capacity for phonology.

Interpretation domains "provide explicit characterizations of linguistically significant properties of sentences, independently of specific formalisms" (Miller, 2001:9). While Miller focuses on theories of syntactic representation, the framework can be adopted nearly wholesale as the comparison of natural classes here is in the same spirit of formalizing this "linguistically significant property" and comparing it across theories via set theoretic tools. If two models map to the same element in some interpretation domain, they are *equivalent* in terms of that domain. Since the domains mean to capture linguistically significant information, the more domains two models are equivalent under ultimately indicates a "stronger" equivalence between them.

Crucially, while unified and v-features are logically equivalent, if we consider their natural class extensions as elements in the interpretation domain for natural class preservation, they are *not* equivalent. However, in the related interpretation domain of contrast preservation, they are equivalent because the set of segmental structures in each map to the same flat set of atomic symbols representing IPA characters. In fact, between just these two domains, there are a number of implications: if two models are natural class preserving, this entails they are contrast preserving. If two models are not contrast preserving, this entails they are not natural class preserving, since the non-equality of basic contrasts will result in at least one natural class extension that will not be present in the other model simply by virtue of one of these contrasts being present in it. What the case study here shows is that it is possible to be contrast preserving while not being natural class preserving.

The utility of framing these distinctions in terms of interpretation domains is that it enumerates exactly what linguistic information may or may not be equivalent between two theories. It might be the case that instances of non-equivalence exist but are not relevant for a specific argument or analysis (for instance, if a theory does not care about natural classes, it does not care that two representations are not natural class preserving). Yet, that does not mean the difference does not exist between the two models. In the discussion of strong generative capacity for morphology, (Dolatian et al., 2021) state the following for phonology:

> In morphology and phonology, there are fewer debates on generative capacity. We speculate that this is due to two issues. First, morphology and phonology have comparatively restrictive WGC [weak generative capacity]. Second, it is unclear what external basis (grounding) should be used

for SGC [strong generative capacity], and thus what diagnostics or metrics to use. (Dolatian et al., 2021:229)

The domain of natural class preservation, along with the weaker notion of contrast preservation, are then two metrics to use, and others can be defined following this same program. With logical transductions serving as the formal foundation, any eventual findings across different domains are both rigorous and analyzable in a theory-independent way, while still respecting the consequences these differences may have in phonological theory as a whole.

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